

# Lund's Tilting Pad Journal Bearing Pad Assembly Method

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*This paper summarizes the development during the last 50 years of tilting pad journal bearing analysis and design. The major impetus of this development was a landmark paper published by Jørgen Lund in 1964, "Spring and Damping Coefficients for the Tilting-Pad Journal Bearing." His paper contained the first widely published dynamic coefficients for tilting pad bearings along with his pad assembly method equations. In the 38 years since Lund's publication, many other authors have written tilting pad journal bearing codes, the first of which were based on Lund's assembly method. These assembly method codes were utilized for many years to analyze and design tilting pad bearings for improved rotordynamic performance. During this time, some key design tools were developed utilizing Lund's method. Other authors have written newer codes which solve the energy and elasticity equations iteratively with the pressure equation, including pad degrees of freedom. With the simple addition of a turbulence correction and heat balance, many designers continue to utilize Lund's method, shunning the more modern codes. [DOI: 10.1115/1.1605767]*

**Keywords:** Jørgen Lund, Tilting Pad Journal Bearing, Pad Assembly Method

## Introduction

The state of the art in tilting pad journal bearing design and analysis has advanced tremendously in the last 50 years, spawned by a landmark paper by Jørgen Lund in 1964 [1]. Lund's paper, "*Spring and Damping Coefficients for the Tilting-Pad Journal Bearing*," was the first major published document that contained tilting pad journal bearing stiffness and damping coefficients. Furthermore, his paper presented the innovative analytical methodology that Lund used to determine these dynamic characteristics. His analytical procedure is commonly known as "Lund's pad assembly method."

Prior to 1964, tilting pad journal bearing studies consisted of steady-state analyses which were limited to determining load capacity and power loss. For many years, the only analysis available was detailed in a 1953 paper by Boyd and Raimondi [2]. A pivoted flat slider on a flat runner was used to determine results which roughly approximated the special case of the bearing assembled clearance equal to the tilting pad machined-in clearance (i.e., zero pad preload). With no knowledge of the dynamic characteristics, they concluded that tilting pad bearings offer "No striking advantages over plain journal bearings . . ." A second paper by the same authors in 1962 expanded their analysis in an attempt to include preloaded pads by approximating the actual pad radius of curvature by adding a crown to the pivoted slider [3].

Lund's pad assembly method calculates the stiffness and damping contribution of each individual pad of a tilting pad journal bearing by considering each pad as a partial arc bearing. "A summation over all pads results in the combined spring and damping coefficients for the complete tilting-pad journal bearing" [1]. The inertia of the pad was included in the analysis. Bearing pad axial length-to-journal diameter ratio ( $L/D$  ratio), pivot loading and pad preload for vertical rotors were all considered. Lund utilized the finite difference method for the partial arc Reynolds equation solution. Stiffness and damping design curves were presented for assembled tilting pad journal bearings with 4, 5, 6 and 12 centrally pivoted pads. Results for the 4 pad bearing were compared to experimental results published in 1958 by Hagg and Sankey [4].

Orcutt [5] extended the work of Lund with the inclusion of

turbulence in a paper published in 1967. Using Lund's pad assembly method, a 4 pad tilting pad journal bearing was analyzed and design plots presented.

A 1969 publication by Elwell and Findlay [6] presented load capacity and power loss data for offset pivoted tilting pad journal bearings of various pad preloads. The authors analyzed a partial arc bearing and performed vector addition to calculate the load capacity and power loss of the complete bearing. The partial arc analysis was based on a 1964 gas bearing paper by Castelli, Stevenson and Gunter [7].

Nicholas, Gunter and Allaire, 1979 [8], employing the finite element method [9] to determine the single pad dynamic data, utilized Lund's pad assembly method to present stiffness and damping design curves for assembled 5 pad tilting pad bearings of varying pad preloads, pivot offsets and pivot load orientations. Nicholas, Gunter and Barrett, 1978 [10], used the data in [8] to show the effects of pad preload, pivot offset and pivot loading on the stability of an 11 stage centrifugal compressor.

Jones and Martin, 1979 [11], also utilized the pad assembly method along with the finite difference method to produce steady state and dynamic properties of 5 pad centrally pivoted bearings including the effects of turbulence. Furthermore, while using an isoviscous solution for the partial arc single pad analysis, their model allowed for different temperatures on each pad. They compared their results to the experimental results of Yamauchi and Someya, 1977 [12] and to the analytical results from [8] and from Shapiro and Colsher, 1977 [13].

Other authors also used Lund's pad assembly method. Among them were Abdual-Wahed, Frene and Nicolas, 1979 [14], who considered fitted pad bearings with the pad radius equal to the journal radius.

In reference [13], the authors discuss the fact that, since the pads of a tilting pad journal bearing tilt, each pad adds a degree of freedom to the journal bearing system. Thus, for a 5 pad bearing, there are 7 degrees of freedom (the  $x, y$  journal motion and the 5 tilt modes of the pads). This results in a  $7 \times 7$  stiffness and a  $7 \times 7$  damping matrix. These  $7 \times 7$  matrices may be reduced to standard  $4 \times 4$  matrices by assuming a pad excitation frequency. The design curves presented in Lund's 1964 paper [1] are based on synchronous frequency as does the reduced data in references [5], [8], [10], [11], [14]. Reference [13] presents two sets of full  $7 \times 7$  stiffness and  $7 \times 7$  damping matrices for a 5 pad bearing with zero and 50% pad preload along with the reduced  $4 \times 4$  data for a

Contributed by the Technical Committee on Vibration and Sound for publication in the Jørgen Lund Special Issue of the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript received June 2003. Guest Associate Editor: R. Gordon Kirk.

synchronous frequency, called synchronously reduced coefficients. The authors also present the equations for reducing full stiffness and damping matrices that include pad tilt degrees of freedom to reduced  $4 \times 4$  matrices.

Other authors [15–19] have investigated the frequency dependency of the reduced tilting pad bearing characteristics starting with Parsell, Allaire and Barrett in 1983 [15] and continuing with Wygant in 2001 [19].

Also starting in the late 1980's, more advanced tilting pad journal bearing codes were developed which did not treat the pads as independent partial arc bearings. Instead, the steady state operating characteristics were determined with a global, fully assembled analysis [20–25].

The first example of this development was presented by Knight and Barrett, 1988 [20]. They solved Reynolds equation using the finite element method for a fully assembled tilting pad bearing assuming a parabolic axial pressure profile. Their methodology includes the solution of a first order energy equation with constant axial and approximate radial temperature profiles. The authors found the journal equilibrium position by iterating on the imposed load, leading edge boundary conditions, journal temperature, pad rotation angles and the coupled pressure (Reynolds) and energy equations.

Branagan, 1988 [21], used similar axial pressure and temperature approximations but included pad and pivot elasticity effects when solving for the dynamic properties of a fully assembled tilting pad journal bearing including pad tilt. Other authors followed in the 1990's solving the Reynolds equation with the energy equation and the elasticity equation for the assembled bearing [22–25]. All three equations are iteratively coupled. For example, Kim, Palazzolo, and Gadangi [24] check the convergence on the pad tilt angles, journal eccentricity, shaft temperature, fluid film temperature, pad temperatures, pad deformations and drain temperature at each iterative step.

Clearly the tilting pad journal bearing analyses from the late 1980's and 1990's are more sophisticated compared to Lund's pad assembly method. However, his method and 1964 paper laid the foundation for these later analyses. Furthermore, for the last 38 years, Lund's method has been successfully utilized by this author [10,26–28] and many others in designing tilting pad journal bearings for the rotating equipment industry.

This paper revisits the pad assembly method by first examining Lund's original work. Next, the application of Lund's method will be examined by reviewing 24 years of this author's tilting pad journal bearing design experience with the pad assembly method. The usefulness and importance of Lund's method concerning turbomachinery rotordynamics will be emphasized.

### Lund's Pad Assembly Method

The pad assembly method calculates the stiffness and damping properties of a single non-tilting pad over a range of eccentricities. The journal seeks its equilibrium position at a specific attitude angle and eccentricity, Fig. 1. The pivot film thickness is calculated and the dynamic properties determined by small perturbations in displacement for stiffness and velocity for damping [1,8].

Next, the contributions to stiffness and damping of each individual pad are determined by calculating the pivot film thickness,  $hp_1$ ,  $hp_2$ , etc., for each pad in an assembled bearing (Fig. 2). The stiffness and damping properties of the assembled bearing are determined by summing all pad contributions using the equations presented in [1,8]. Contributions from the unloaded pads are set to zero [1].

Sample design plots from Lund's paper are shown in Figs. 3 (between pivot loading) and 4 (on pivot loading) for a 5 pad assembled bearing. Note that  $K_{xx}$ ,  $K_{yy}$ ,  $C_{xx}$  and  $C_{yy}$  are shown on the plots. The figures "... do not include cross-coupling terms  $K_{xy}$ ,  $K_{yx}$ ,  $C_{xy}$  and  $C_{yx}$  because they vanish when the pad inertia is neglected" [1]. This was the first time that this important fact was identified, thus implying a substantial stability advantage over

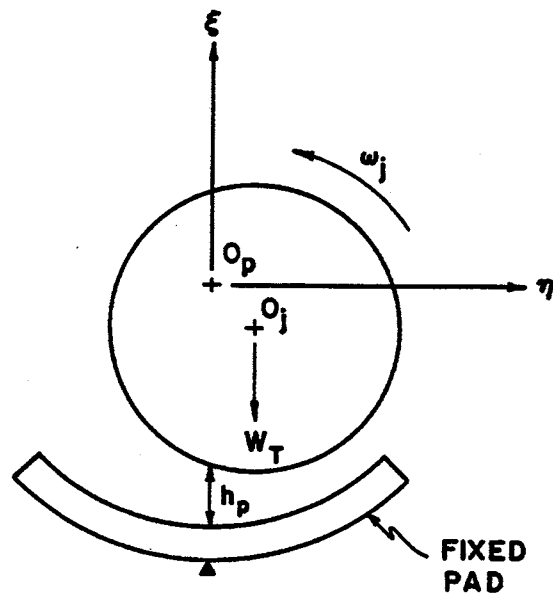


Fig. 1 Single pad schematic

fixed geometry bearings. However, since his dynamic data is presented as 8 stiffness and damping coefficient curves, it does not reflect their frequency dependency. Thus, the coefficients are "reduced" [13] assuming "... harmonic motion ..." [1] which Lund set to synchronous frequency. However, his assembly method is general enough to accept non-synchronous excitation frequencies without additional complexities.

Lund also includes a critical mass curve on his plots which can be used to determine "... The onset of pad resonance ..." which he defined as "... the speed at which the phase angle between a zero-inertia pad and a pad with finite inertia becomes  $90^\circ$ " [1]. Curiously in Fig. 4 (load on pivot), Lund's stiffness curves tend toward zero at a Sommerfeld number,  $S$ , of around 10. Lund describes this phenomena briefly. "The most unusual aspect of the theoretical results is the sudden reduction in the direct-coupled spring coefficients  $K_{xx}$  and  $K_{yy}$  when approaching pad resonance" [1].

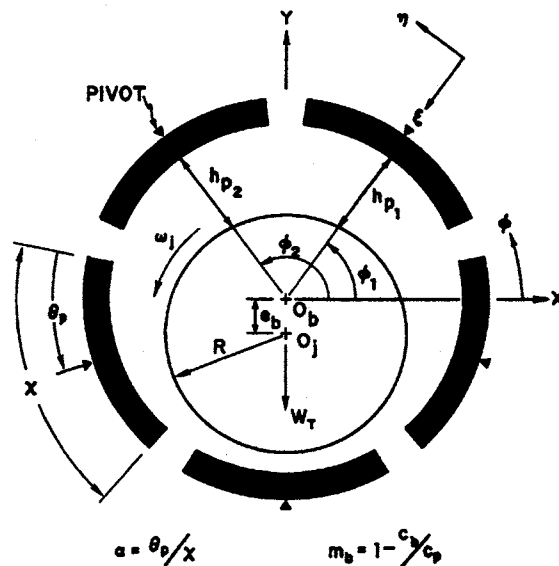


Fig. 2 Tilting pad bearing schematic

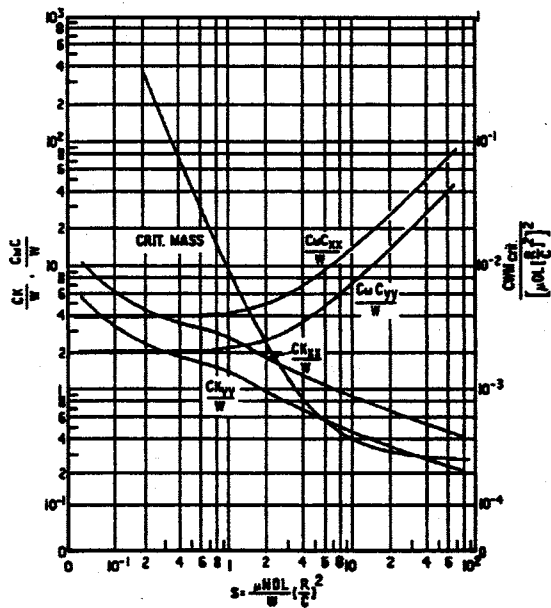


Fig. 3 Lund's 5 pad load between pivot data

Lund compares his theoretical data to some experimental data from reference [4]. His comparison plot for a load between pivot 4 pad bearing is included here as Fig. 5. It should be noted that Lund's coordinate system is defined as: y-axis horizontal to the right, x-axis vertically downward, counter-clockwise rotation.

Further examples of pad assembly method results from other authors are illustrated in Figs. 6-9 with a coordinate system defined in Figs. 1 and 2. Sample single pad data curves are shown in Fig. 6 [8]. Single pad data is a function of bearing  $L/D$  ratio, pad arc length, pivot offset and pad excitation frequency. This assumes that pad inertia is neglected which appears to be a good assumption [13]. Figure 6 shows curves for 50% centrally pivoted and 55% offset pivoted single pads. Figure 7 compares load on pivot to load between pivots for a 5 pad tilting pad bearing with zero pad preload and centrally pivoted pads [8].

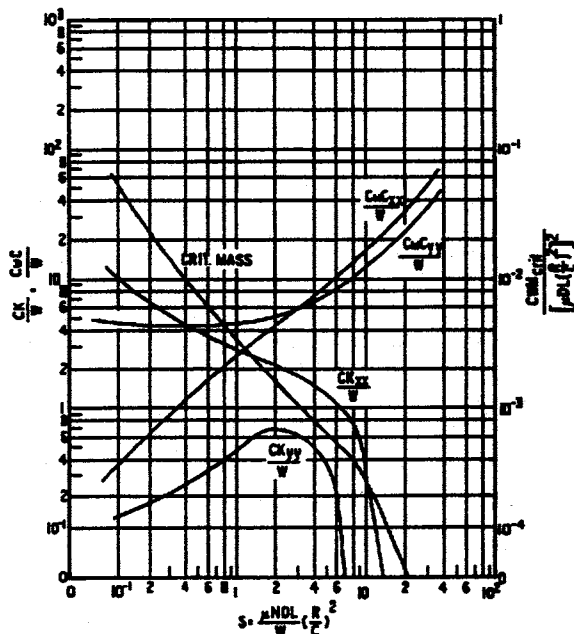


Fig. 4 Lund's 5 pad load on pivot data

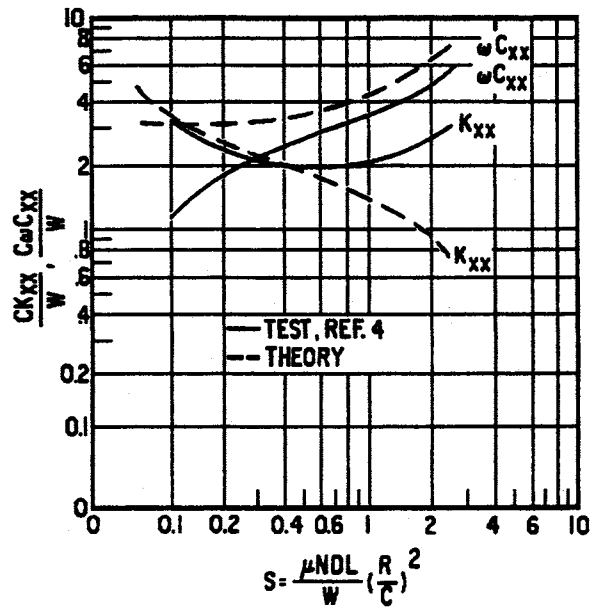


Fig. 5 Lund's data vs experimental

Figure 8 compares assembled synchronously reduced theoretical data from references [8], [11] and [13]. All 3 independent data sources produced essentially the same results. A slight error is evident in reference [8] for damping at high Sommerfeld numbers as the authors did not neglect damping for the unloaded pads. Figure 9 shows a reasonable comparison of assembled synchronously reduced theoretical data from reference [11] to experimental data from reference [12].

### Rotordynamic Implications of Lund's Pad Assembly Method

Armed with Lund's pad assembly method for determining tilting pad journal bearing stiffness and damping coefficients, rotordynamicists in the late 1970's and the 1980's were finally able to address instability problems similar to the one illustrated in Fig. 10, Gunter, Barrett and Allaire [29], with a reasonable degree of

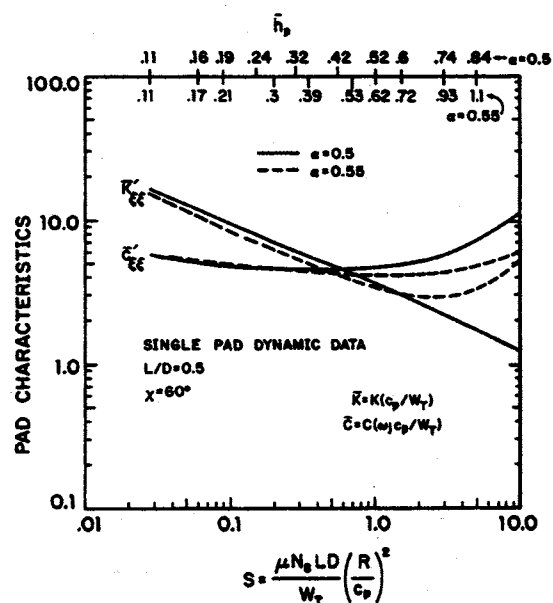


Fig. 6 Single pad dynamic data

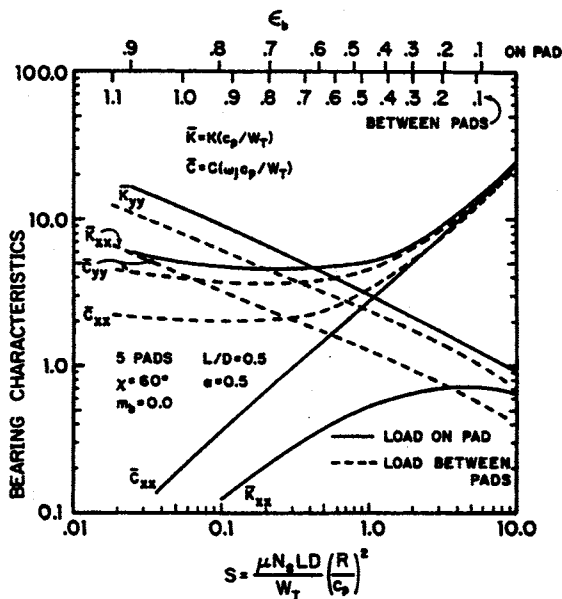


Fig. 7 Effect of pivot loading, 5 pad bearing

confidence. Figure 10 shows a typical centrifugal compressor instability. This compressor is running on tilting pad journal bearings.

Figure 11 shows a critical speed map for an eleven stage centrifugal compressor [10]. In an effort to improve the compressors stability characteristics, several tilting pad bearing stiffness curves are over-plotted on the map. From this figure, it may be tempting to use bearing numbers 5 or 8, the heavily preloaded, high stiffness bearings, and operate below the rotors second critical speed. Conversely, the lighter preloaded bearings with decreased bearing stiffness drops the second critical to the operating speed range.

The prevailing bearing design philosophy for many designers in the 1960's and early 1970's was to increase the bearing's stiffness and damping properties to reduce both synchronous and subsynchronous vibration. The following conclusion is perhaps typical of the bearing design philosophy of the era [5]. "Preloading the tilting-pad bearing results in greater stiffness and damping thus improving dynamic characteristics. . . . Fractional frequency whirl has been observed in the unpreloaded tilting-pad bearing at high speeds and very light loads. . . . Preloading can stop instability completely."

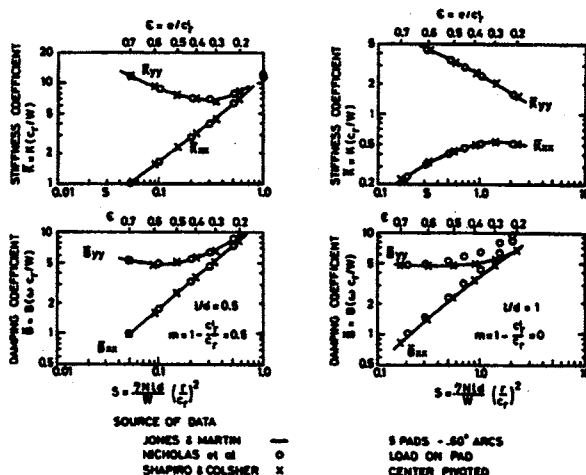


Fig. 8 Jones & Martin vs Nicholas vs Shapiro

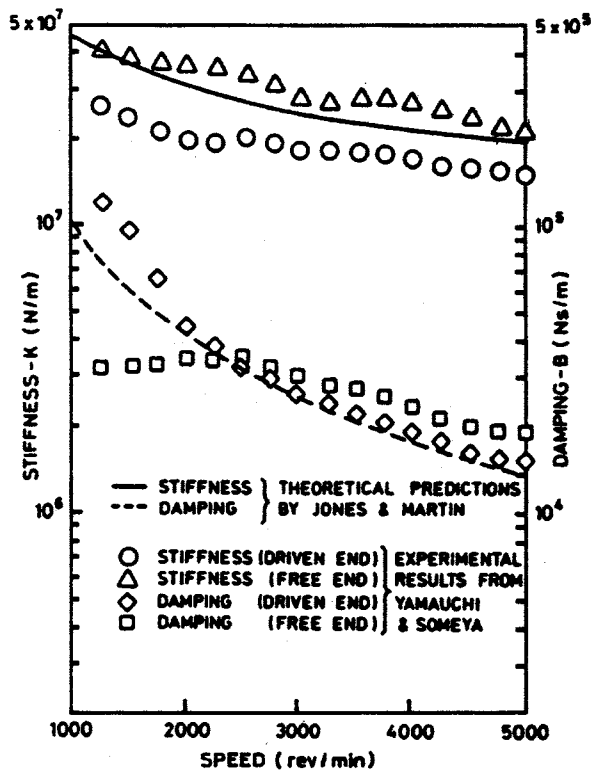


Fig. 9 Jones & Martin vs experimental

As it turns out for many applications, decreasing pad preload decreases bearing stiffness but increases effective damping thereby providing improved stability performance and an overdamped non-responsive second critical speed [10]. This trend may be observed in the stability map shown in Fig. 12. Clearly, as preload decreases, the instability threshold speed (the speed above which the compressor becomes unstable) increases.

The reason for this trend can be seen in Fig. 13, the mode shape plots for the 11 stage compressor with a zero preloaded tilting pad bearing. Decreasing bearing stiffness allows more shaft motion at

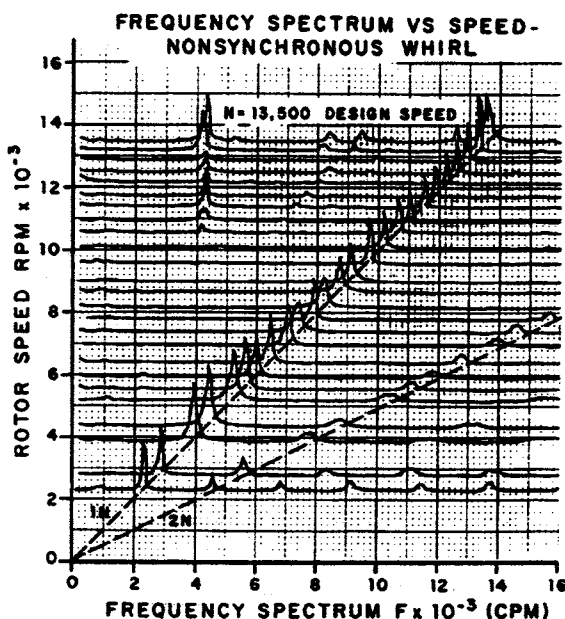


Fig. 10 Centrifugal compressor instability

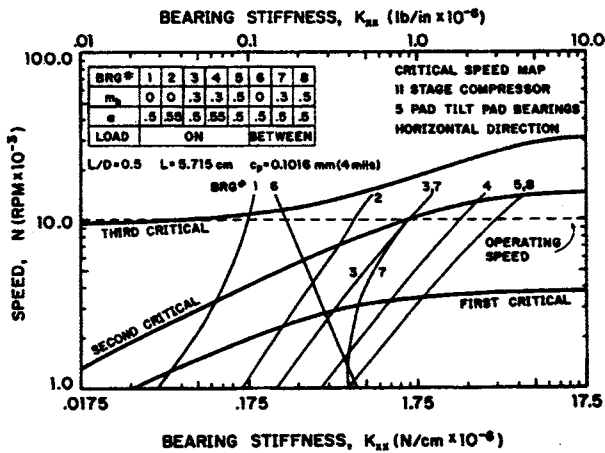


Fig. 11 Critical speed map, 11 stage centrifugal compressor

the bearing locations thereby allowing the bearing damping to become more effective in vibration suppression. In this case, the first mode is nearly a rigid body mode with considerable rotor motion at the bearings. Conversely, increasing both bearing stiffness and damping by increasing pad preload causes less motion at the bearings thereby decreasing effective damping even though bearing damping increased.

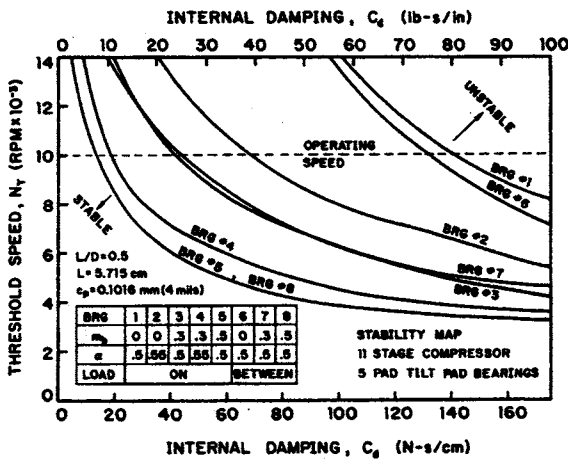


Fig. 12 Stability map, 11 stage centrifugal compressor

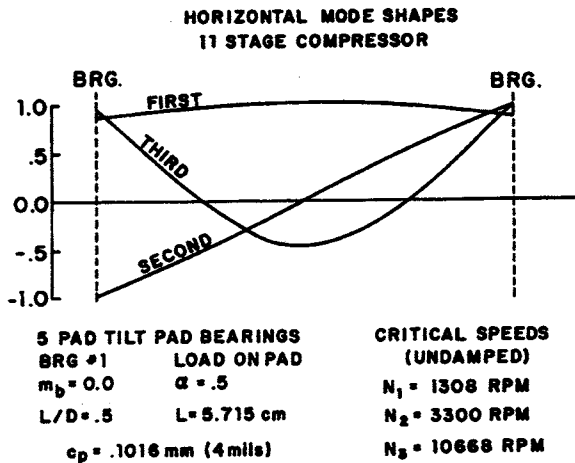


Fig. 13 Mode shapes, 11 stage centrifugal compressor

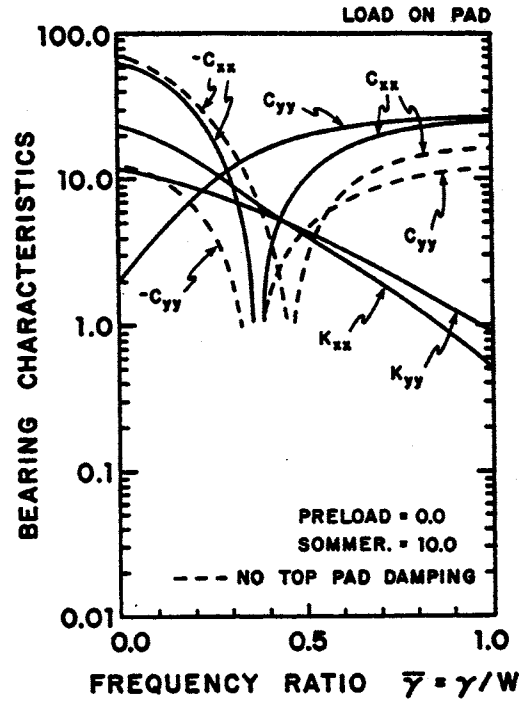


Fig. 14 Frequency dependent data,  $S=10.0$ ,  $m_b=0.0$

Thus, Lund's pad assembly method was directly responsible for a tilting pad bearing design philosophy that provided improved rotordynamic performance for turbomachinery designed and manufactured after the mid to late 1970's.

As a word of caution, there are numerous reasons to reduce pad preload carefully, Nicholas, 1994 [28]. Additionally, the effects shown in Figs. 11 and 12 due to preload changes are somewhat exaggerated as the pad bore was held constant and the preload decreased by increasing bearing assembled clearance. It would be more appropriate to hold the bearing assembled clearance constant and then decrease preload by decreasing the machined-in pad bore clearance. This subject is addressed in detail by McHugh in his discussion of Orcutt [5].

### Tilting Pad Frequency Dependency

The bearing characteristics utilized in the stability plot of Fig. 12 are synchronously reduced. Again, this was the prevailing stability methodology at the time. A quote from reference [13], 1977, from a paragraph concerning reducing a  $7 \times 7$  matrix to a  $4 \times 4$  illustrates this attitude. "An assumption must be made as to the vibrating frequency of the pads . . . . In general, synchronous frequencies are selected."

When this author presented the results of Fig. 12 in 1978 [10], Lund pointed out that it is mathematically incorrect to use a synchronous frequency for a stability calculation. He went on to say that the damped natural frequency should be used instead. These comments by Lund triggered a huge proliferation of research that is still ongoing today concerning the frequency dependency of tilting pad journal bearing characteristics [15-19].

The plot shown in Fig. 14 from Parsell, Allaire and Barrett [15] caused quite a stir when it was published in 1983. Could the bearing damping tend toward zero for pad frequencies around 0.5 (50% of synchronous frequency) as the plot suggests? For zero preload and very high Sommerfeld numbers ( $S=10.0$ ), the answer is theoretically yes. Figure 15 shows that the bearing damping has a much more gradual decrease and the stiffness a more gradual increase for reasonable preloads (30%) and a Sommerfeld number ( $S=1.0$ ) that is more representative of industrial turbomachinery.

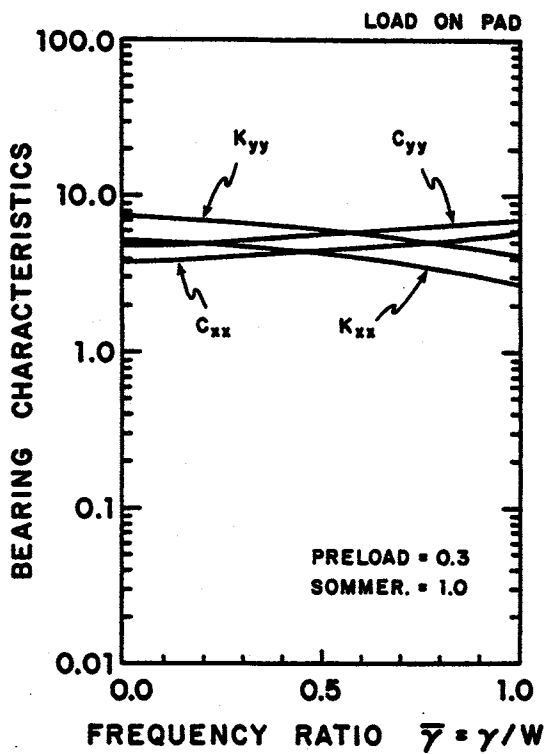


Fig. 15 Frequency dependent data,  $S=1.0$ ,  $m_b=0.3$

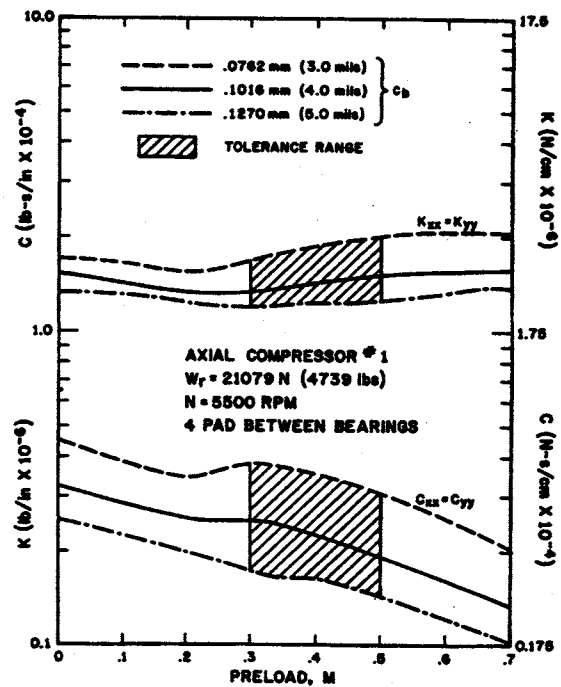


Fig. 16 Stiffness & damping vs preload & bearing clearance, 4 pad bearing

### Further Pad Assembly Method Implications

While the influence of pad vibration frequency on tilting pad bearing dynamic coefficients was acknowledged and became an area of study, synchronously reduced coefficients remained the design standard for much of the turbomachinery industry from the late 1970's through the early 1990's. Using Lund's pad assembly method with reduced coefficients, many tilting pad bearing design guidelines were developed during this time. Several examples are presented in Figs. 16 through 18.

Figure 16 [26,27] illustrates the effects of pad preload for an axial compressor. As preload decreases, the synchronously reduced stiffness remains about the same but damping increases. Thus, as before, lowering preload increases effective damping. Conversely, as the bearing assembled clearance decreases, stiffness and damping both increase, decreasing effective damping.

Figure 17 [26,27] shows the same preload effect for another axial compressor. Additionally, as pad axial length,  $L$ , increases (bearing  $L/D$  ratio increases), the bearing damping increases and bearing stiffness decreases. Both effects combine to increase effective damping. Increasing tilting pad bearing  $L/D$  ratios has become a powerful design tool, often times useful in decreasing rotor vibration.

Another example of tilting pad bearing optimization for improved stability performance is presented in Fig. 18 for a 2 stage centrifugal compressor [26]. The compressor's stability characteristics are improved considerably by optimizing the pad preload and assembled clearance utilizing the pad assembly method and synchronous reduced coefficients. Once again, lowering pad preload improves stability.

Clearly, Lund's pad assembly method, which was utilized in Figs. 16–18, allowed bearing designers to gain tremendous insight into tilting pad journal bearing design and optimization in the late 1970's and 1980's. Many design guidelines developed in this era are still valid today such as longer pad lengths and lower pad preloads to increase effective damping.

Lund's pad assembly method is still being used today to successfully design tilting pad journal bearings in the rotating equip-

ment industry. With the addition of a turbulence correction, a heat balance used to predict bearing operating temperatures [28] plus pad [30] and pivot flexibility [31], his method remains popular with many bearing designers, this author included.

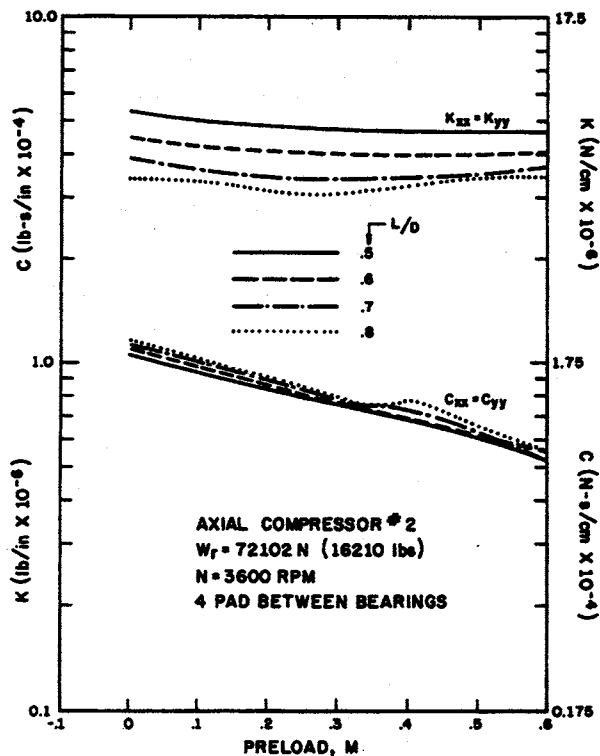


Fig. 17 Stiffness & damping vs preload &  $L/D$  ratio, 4 pad bearing

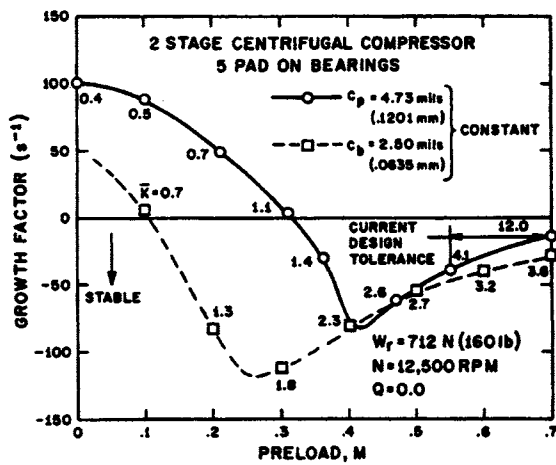


Fig. 18 Stability performance vs preload & bearing clearance, 2-stage centrifugal compressor

### Conclusions

In the last 50 years, the state of the art in tilting pad journal bearing design and analysis has advanced from a steady-state plane-pivoted flat slider bearing methodology to three dimensional thermo-elasto-hydrodynamic solutions. The impetus to this analytical development was the landmark paper by Lund in 1964 concerning his pad assembly method. His paper contained the first widely published stiffness and damping coefficients for tilting pad journal bearings along with his analytical procedure.

Many researchers, including this author, adopted his assembly method to write tilting pad bearing dynamic computer programs. These codes were utilized for many years by a significant number of bearing designers to analyze, optimize and design tilting pad journal bearings for improved rotordynamic performance. During this time, some key design tools were developed utilizing Lund's method such as increasing pad axial length and reducing pad preload to increase effective damping.

Lund's pad assembly method is still being used today to design tilting pad journal bearings. His method offers the advantage of reduced run times and avoids the iteratively coupled solutions and possible non-convergence problems of the newer thermo-elasto-hydrodynamic methodology. Lund's assembly method has stood the test of time for almost four decades. There is reason to believe that it may just continue to be utilized well into the 21st Century.

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